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Spatially Optimal Steady State Phosphorus Policies in Crop Production

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Abstract
We analyze optimal phosphorus fertilization and erosion control policies in a spatial, dynamic, stationary framework. First-best instruments to incentivize farmers to undertake the socially optimal choices are analyzed both analytically and empirically. The empirical application is conducted for a cereal production area of 4 hectares. We find that taxes on phosphorus use can equivalently be levied either on fertilizer use or directly on soil phosphorus. However, tax on soil phosphorus is simpler and poses lower information requirements for the social planner. Also, the potential differences in socially and privately applied discount rates are shown to affect optimal tax rates substantially.

Keywords: Phosphorus fertilization, Phosphorus runoff, Dynamic programming, Steady state, Pigouvian tax
1 Introduction

Eutrophication of surface waters is a major environmental concern worldwide. As point sources have been able to cut down nutrient emissions, the focus of abatement policies has moved to nonpoint sources, especially agriculture. In the Baltic Sea catchment area, for example, agriculture and managed forestry are estimated to account for 57% (68%) of the anthropogenic phosphorus (nitrogen) loading entering inland surface waters (HELCOM, 2004). So far, only little progress has been made in tackling this problem. Reductions in phosphorus loads would be essential in particular when improving the water quality of many inland waters.

In arable lands, phosphorus (together with other nutrients) accelerates plant growth. Crop yield response to phosphorus comprises the responses to potentially plant available soil phosphorus reserves (henceforth: soil phosphorus) and of the direct response to phosphorus fertilizer application. The soil phosphorus, which clearly dominates the yield response, can be accumulated or depleted only slowly. Choosing the annual phosphorus application levels is therefore a dynamic problem.

Unfortunately, increasing the level of soil phosphorus also tends to increase the phosphorus loss to surface waters, enhancing primary production in form of harmful algae growth. Phosphorus loss consists of two forms which differ significantly in their runoff, abatement and damage characteristics. The loss of dissolved phosphorus (DP) is largely determined by the level of soil phosphorus. Hence, soil phosphorus is a direct source of social benefits and damages. It is readily bioavailable and contributes directly to eutrophication in the receiving waters. The loss of particulate phosphorus (PP), on the other hand, is driven mainly by erosion and only part of it will contribute to eutrophication.\(^1\) Erosion susceptibility varies spatially along with, for instance, the slope of a field parcel. Hence, optimal phosphorus control should acknowledge spatial and temporal aspects.

In this study, we analyze three questions related to efficient, stationary phosphorus policies. Firstly, what is the socially optimal stationary level of mineral phosphorus fertilization in cereal production? There is a trade off between the profits from crop production and damages due to DP

\(^1\) The desorption processes are influenced by the conditions of the water body. High pH, high soil to solution ratio or low oxygen levels, for example, favor the desorption processes.
loss as both are influenced by the same variable: soil phosphorus. Secondly, how should optimal erosion control policy acknowledge the spatial heterogeneity in field slopes and upslope field lengths? The steeper the slope, the higher the susceptibility for erosion is. However, even for fields with identical slopes and sizes the runoff may vary: it makes a difference whether the upslope field is 50 or 200 meters. Thirdly, how can we characterize alternative tax-subsidy instruments to induce the socially optimal outcomes? How does the special character of phosphorus as a direct source of benefits and damages affect the instrument design?

To answer these questions, we postulate a dynamic programming model for optimal use and abatement of phosphorus. We analyze the steady state conditions for private and social optima, and the instruments to induce the latter. Specifically, we consider two alternative instrument bases, and the effect of the discount rate on first-best taxes. Finally, we conduct an empirical application for a cereal production area of 4 hectares. Agricultural runoff is a non-point externality for which the point source based analysis is not directly applicable. However, by modeling the generation of externalities with a continuously differentiable function whose arguments are (partly) the same as in the production function, we can analyze the issue with standard methods (Griffin and Bromley, 1982).

There are only few studies analyzing the dynamic phosphorus control. Schnitkey and Miranda (1993) conduct a steady state analysis on controlling phosphorus runoff from livestock producing farms with crop production. They solve the privately optimal manure application radius around the livestock facility. Inside this radius the farmer applies only manure, outside only commercial fertilizer. They do not, however, consider the interlinkages of DP loss and excessively high soil phosphorus levels, but instead assume (uniform) erosion as the only source of phosphorus losses.

Goetz and Keusch (2005) conduct a dynamic phosphorus policy analysis, but they also consider soil erosion as the only source of phosphorus loss. Particulate phosphorus in eroded material, however, has a relatively low bioavailability. Therefore, its role in eutrophication is not proportional to its share of the total phosphorus loss. More importantly, PP loss is not a distinctively dynamic phenomenon, whereas DP loss is.

The analysis on alternative instrument basis has focused on assessing second-best instruments applicable for agri-environmental programs. Larson et al. (1996), among others, analyze the efficiency of instruments levied on alternative inputs. The key requirement for such inputs is
correlation with water quality (Horan and Shortle, 2001). In this study, the alternative tax bases differ in a dynamic sense. The idea is novel in agri-environmental literature and we present it in a first-best context.

To our knowledge, there have been no studies that systematically combine the edaphic soil phosphorus dynamics, hydrological phosphorus loss processes and economic, dynamic decision making on phosphorus fertilization. We characterize dynamically optimal stationary phosphorus policies, and characterize alternative instruments to sustain the steady state.

The rest of the study is organized as follows. In the following section we present the analytical model and derive analytical results on first-best instruments. Section 3 presents the empirical application. Section 4 presents the results and the last section concludes.

2 The analytical model

To economic problem of the decision maker (either the social planner or the private farmer) is to maximize welfare over an infinite time horizon by choosing phosphorus application rates and vegetative filter strip (VFS) widths at each period for a parcel of land. The welfare consists of revenues from crop production minus the environmental damage from phosphorus loss. The choice of other nutrients as well as other variable inputs is assumed fixed. Hence, phosphorus fertilization and VFS width are the control variables and the state variable is the potentially plant available soil phosphorus, approximated by soil test phosphorus measure (STP)\(^2\). The damage is due to annual phosphorus loss consisting of PP and DP losses, where the former is weighted according to its bioavailability. The optimization problem is:

\[
\max_{x_i, b_i} \sum_{t=0}^{\infty} \beta^t \left[ \left( pY(s_t, x_t) - wx_t - FC \right) \left( 1 - A(b_i) \right) - C(A(b_i)) - D^i \left( L(s_t, b_i) \right) \right] \\
= \max_{x_i, b_i} \sum_{t=0}^{\infty} \beta^t \left[ \pi_t - D^i \left( L(s_t, b_i) \right) \right] = \max_{x_i, b_i} \sum_{t=0}^{\infty} \beta^t \left[ \Pi^t \right] \\
s.t. \quad s_{t+1} = \Gamma(s_t, x_t),
\]

with complementary conditions \(x, b \geq 0\).

\(^2\) In Finland, the test is based on extraction with acid ammonium acetate (Vuorinen and Mäkitie, 1955).
The discount factor \((\beta^i, i = \{s,p\} = \{social~planner,~private~farmer\})\) is derived from the market discount rate or from the social rate of time preference: \(\beta = \frac{1}{1+r}\). Both are constant in time but not necessarily equal. The one period revenue comprises of the deterministic crop yield \((Y)\) times the constant price \((p)\) minus the costs of phosphorus fertilization \((wx)\) and other fixed and variable costs \((FC)\). The VFS width \((b)\) determines the area of land set aside of production \((A)\) as well as the operation and maintenance costs for the VFS \((C)\). These components constitute the one period private profit \((\pi)\). The last component of the one period welfare is the damage \((D)\) due to phosphorus loss \((L)\) which is a function of the STP-level \((s)\) and VFS width. We assume that it does not affect private farmer’s welfare, i.e. \(D^p = 0\). Transition function \((\Gamma)\) has only fertilizer use and previous period’s STP level as its arguments. For brevity, we sometimes use \(\Pi^i\) to denote the one period welfare.

The partial derivatives of the phosphorus response function are assumed to be: \(Y_s > 0; Y_{ss} < 0; Y_x > 0\) and \(Y_{sx} < 0\). Referring to Saarela et al. (1995) we assume: \(Y_{sx} < 0\). The partial derivatives of \(\Gamma\) are assumed to be: \(\Gamma_s > 0; \Gamma_{ss} < 0; \Gamma_x > 0; \Gamma_{sx} < 0; \) and \(\Gamma_{sx} < 0\). The phosphorus loss is increasing and convex in \(s\), and decreasing and convex in VFS width; \(L_s > 0; L_{ss} > 0; L_b < 0; L_{bb} > 0\).

### 2.1 Privately and socially optimal steady state choices

The optimality conditions for (1) can be derived from the steady state Bellman equation:

\[
V(s) = \max_{s,b} \left[ \Pi^i(s,x,b) + \beta^i V(\Gamma(s,x),b) \right],
\]

At the steady state, the choices and the state remain unchanged between the periods. The optimality conditions for decision maker \(i\) consist of two Euler equations and the stationary condition, combined with a complementary condition:
The Euler equations characterizing the optimal stationary use of phosphorus fertilizer and the choice of VFS width \( b \) are given in equations (3a) and (3b), respectively. Note that the partial derivative \( (\partial \bar{i}_b) \) is zero. Further, assuming that \( \beta' > 0 \) and \( 0 < \Gamma < 1 \), the optimal VFS width is found at \( \Pi'_b = 0 \). That is, at the optimum the static marginal costs from VFS are equal with its marginal benefits.

Table 1 presents side by side the optimality conditions (3a - 3c) for the private farmer and for the social planner. To facilitate discussion, we open and rearrange the terms:

**Table 1. The stationary optimality conditions.**

<table>
<thead>
<tr>
<th>Private</th>
<th>Social</th>
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<tbody>
<tr>
<td>( \pi_x + \beta'' \Gamma_x \frac{\pi_x}{(1 - \beta' \Gamma_x)} = 0 )</td>
<td>( \pi_x + \beta' \Gamma_x \frac{(\pi_x - D_x)}{(1 - \beta' \Gamma_x)} = 0 ) (4a)</td>
</tr>
<tr>
<td>( \pi_b = 0 )</td>
<td>( \pi_b - D_b = 0 )       (5a)</td>
</tr>
<tr>
<td>( s = \Gamma )</td>
<td>( s = \Gamma )           (6a)</td>
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At the optimum, the private farmer balances the marginal effects for current period profits \( (\pi_x) \) and the discounted sum of future profits (the second term in 4a). The value (shadow price, imputed value) of a marginal unit of soil phosphorus is given by \( \frac{\pi_x}{(1 - \beta' \Gamma_x)} \). It is the sum of an infinite geometric sequence of profits generated by a marginal increase in soil phosphorus \( (\pi_x + \beta'' \Gamma_x, \pi_x + (\beta'' \Gamma_x)^2 \pi_x + ...) \). The marginal increase is generated by the marginal effect that the previous period's fertilization has on soil phosphorus \( (\Gamma_x) \). Because the value is gained in the following period, the sum has to be discounted back one period.
All components of the second term in (4a) are positive. Hence, to satisfy the condition, the marginal profit in the current period ($\pi_\gamma$) has to be negative at the optimum. Due to concave phosphorus response (and hence profit) function this means that at the dynamic steady state optimum the farmer uses more fertilizers than he/she would be using if optimizing on static grounds. Static optimum would imply $\pi_\gamma = 0$.

The optimal VFS width (5a and 5b), on the other hand, is chosen on static grounds. It is assumed to have no dynamic elements (no effect on either STP or DP abatement). For the private farmer the marginal profit from constructing VFSs (LHS in 5a) is negative for all $b$. Hence, the complementary condition becomes active: the privately optimal VFS width is 0.

The social optimum acknowledges that increasing soil phosphorus increases both the profits and the environmental damage. This alters the condition (4a). In social optimum (4b), the imputed value of soil phosphorus is $\pi_\gamma - \frac{D_s}{(1 - \beta^s T_s)}$. The optimality condition balances the marginal effects for this and future periods. Because the marginal damage ($D_s$) is positive for all $s$, the second term is smaller in the social optimum (4b) than at the private optimum (4a). This implies that at the social optimum, the immediate marginal profit ($\pi_\gamma$) is a smaller negative number than at the private optimum. Due to concavity this implies that the use of fertilizers is lower under the social optimum than under the private optimum.

At the social optimum, the marginal costs from increasing the VFS width equals its marginal benefits; $\pi_b = D_b$. The complementary condition holds if the marginal cost of first VFS width unit is higher than its marginal benefit. Otherwise, the socially optimal VFS width is strictly positive. For both private and social optimum, conditions 6a and 6b guarantee the stationarity of the solution.

These differences in optimality conditions imply that the actions taken by farmers generate an externality for the society. To internalize this, the social planner may pose economic instruments on farmers' actions.

### 2.2 Instrument design
We examine two Pigouvian first-best tax-subsidy schemes with alternative phosphorus tax bases. Both schemes incentivize the private farmer to undertake socially optimal stationary actions. The tax is based either on the use of phosphorus fertilizers \((x)\) or on STP level \((s)\). Posing a Pigouvian tax on STP is a novel idea, not examined in the previous literature. The subsidy is based on VFS width. Under both schemes, the one period profit becomes \(\pi^r = \pi - \tau(x) + \nu(b)\), where \(\tau(x)\) refers to a phosphorus tax on either of the bases and \(\nu(b)\) refers to a VFS subsidy.

The social planner chooses a tax to equalize conditions (4a) and (4b), and a subsidy to equalize (5a) and (5b). The assumptions on the first and second derivatives of the relevant functions guarantee that there is a unique VFS - phosphorus application combination for any parcel type. We first present the VFS subsidy which is a common component for both schemes. The marginal VFS subsidy must satisfy:

\[
\pi_v^s - D_v^s = \pi_v + \nu_v \iff \nu_v = -D_v.
\]  

(7)

From \(D_v < 0\) it follows that \(\nu_v > 0\), i.e. it is a subsidy. Equation (7) defines the marginal VFS subsidy which is at the steady state optimum unique for each parcel. There are, however, infinitely many alternative subsidy functions that satisfy (7).

The fertilizer-based tax must be adjusted to make the pair of equations in (8) identical, i.e.:

\[
\pi^r_{x}(1 - \beta^p \Gamma_s) + \beta^p \pi_{x} \Gamma_s = \pi^r_s(1 - \beta^s \Gamma_x) + \beta^s (\pi_x - D_x) \Gamma_x
\]  

(8)

where \(\pi^r_{x}\) is the marginal profit w.r.t. fertilization: \(\pi^r_{x} = \pi_x - \tau_x\), and \(\tau_x\) is the marginal tax. Assume first that the socially and privately applied discount factors are not identical \((\beta_s > \beta_p)\). Solving for the marginal tax rate \((\tau_x)\) yields (for tractability, we denote parts of the function with new symbols):

\[
\tau_x = \pi_x \left[ \frac{64 7 48}{1 4 44 4 4 4 4 4}\right] - \left[ \beta^p - \beta^p \right] \frac{64 7 48}{1 4 44 4 4 4 4 4} \pi_x \Gamma_s \left( 1 - \frac{\beta^p \Gamma_s}{1 4 44 4 4 4 4 4} \right) + \frac{\beta^s (\pi_x - D_x) \Gamma_x}{1 4 44 4 4 4 4 4}\]  

(9)
The marginal tax depends on the transition function \((\Gamma)\), the profit function \((\pi)\), the private and social discount factors \((\beta^{p,s})\), and on the damage function \((D)\). As the difference in discount rates increases, the difference \((\hat{A})\) becomes a larger negative number. We have shown earlier that \(\pi_s < 0\) at the optimum, hence the overall effect of the first term on tax is decreasing. That is, if private farmers are less patient than the social planner, their privately optimal choices draw closer to the social optimum and the level of the optimal marginal tax decreases. On the other hand, higher discount rate applied by the farmer makes \((\pi_s)\) a smaller negative number which offsets this effect. In any case, the first term \((\hat{B})\) remains positive and therefore lowers the optimal marginal tax.

Also the second term \((\hat{C})\) lowers the marginal tax. It is a product of three terms: the difference in the discount rates, the marginal effect of fertilisation on next period's soil phosphorus, and the imputed value of this marginal change. The two first are zero if the applied discount factors coincide.

The last term decreases as we increase the difference of the discount rates. That is, all terms in (9) operate in the same direction as the difference increases. It may be that the terms \((\hat{B})\) and \((\hat{C})\) outweigh the term \((\hat{D})\) if the difference is high enough. In this case the tax becomes a subsidy on phosphorus fertilization, i.e., the privately optimal level of phosphorus fertilization is lower than the socially optimal level. We can think that the difference in the discount rates is akin to an externality. If this effect outweighs the environmental damage, the instrument should foster input use.

If the socially and privately applied discount factors are identical, (9) simplifies to:

\[
\tau_{s|\beta^{p} = \beta^{s}} = \beta \Gamma_s \frac{D_s}{(1 - \beta \Gamma_s)}
\]  

(10)

The optimal fertilizer tax is a product of two terms. The marginal effect of fertilizer use on soil phosphorus \((\Gamma_s)\) multiplied with the imputed damage caused by a marginal increase in soil phosphorus \(\frac{D_s}{(1 - \beta \Gamma_s)}\). This product is discounted one period.
The tax is increasing in $\Gamma_s, \Gamma_t, \beta$ and $D_t$. The more the phosphorus fertilization contributes to following period's STP, the higher the tax; and the higher proportion of STP is carried over to the next period, the higher the tax. Increase in the discount factor increases the privately optimal STP. This increases phosphorus loss and thereby the environmental damage, i.e. the tax must be increased. Trivially, the tax is zero if the marginal environmental damage w.r.t. $s$ is zero.

The alternative way is to pose the tax on STP directly. Acknowledging that: $\pi_s^* = \pi_s - \tau_s$ we obtain:

$$
\tau_s = \frac{\beta_s}{\beta_p} D_t - \frac{\Gamma_s (\beta_p - \beta_s)}{\beta_p \Gamma_s} - \frac{\Gamma_s (\beta_s - \beta_p)}{\beta_p \Gamma_s}
$$

As in (9), an increase in the difference in discount factors decreases the level of all terms of (11). Setting the discount factors equal simplifies (11) to:

$$
\tau_s|\beta_p = \beta_s = D_s
$$

This is a classical result in environmental economics: at the social optimum, the marginal tax on input must equal its marginal damage.

We can collect the preceding discussion into two propositions on tax instruments for stationary phosphorus control.

**Proposition 1.** The first-best tax on phosphorus control can be based either on phosphorus application or on soil phosphorus. The former implies a tax similar to tax on stock pollutant; the latter a classical tax on 'static' externality.

Proposition 1 embodies an important implication for phosphorus policies. The social planner's information requirements for the tax based on soil phosphorus are lower than for the tax based on phosphorus application. For the former, we need knowledge on the phosphorus loss function and on the environmental damage. For the latter, we also need to be able to calculate the imputed value of soil phosphorus. That is, defining the correct level for this tax requires knowing the soil phosphorus carryover function (and its derivatives). If the tax is based on soil phosphorus, we do not need this
information. Taxes under both bases are uniform if there is no heterogeneity in STP's contribution to damage.

Most of practical environmental policy is conducted via prespecified targets, without defining the environmental damage. In this case, the social planner seeks not the social optimum but the cost-minimizing policy that satisfies the given environmental targets. Here, too, conditioning the policies on soil phosphorus requires less information from the social planner. A prespecified runoff target has a unique counterpart in the level of soil phosphorus. The farmer would be induced to maintain this level if faced by a marginal tax that would be, for instance, zero for values at or below the target level, and extremely high for levels above the target. If such an instrument would be based on phosphorus fertilization, one would also need to know the carryover dynamics to determine the level of annual phosphorus application that corresponds to the desired level of soil phosphorus.

Summa summarum, we can base a phosphorus tax equivalently well on soil phosphorus or on annual phosphorus fertilization. However, if information is costly we would recommend taxes to be based on soil phosphorus.

The implications of the differences in discount factors are summarized in proposition 2:

**Proposition 2.** A difference in discount rates of the private farmer and the social planner affects the level of a first-best phosphorus tax. High enough a difference reverses the tax into a subsidy on phosphorus fertilization.

This proposition has an interesting implication. If the environmental damage depends only on the flow of bioavailable phosphorus; and if the social planner is sufficiently more patient than farmers, there might be no need for regulation\(^3\). In this case, the farmers' lower motivation for investing into soil phosphorus leads to stationary soil phosphorus levels at or below the social optimum. The literature on whether the social planner should apply discount rates at or below the market rate of discount, is abundant. Indisputably, choosing the social planner's discount rate is a matter of subjective evaluation of different generations' welfare. In the aftermath of the Stern Review the

\(^3\) Whether the accumulated stock or the annual flow of phosphorus dominates the generation/acceleration of primary production, is determined by the hydrological conditions. In one extreme we have the shallow lake type with the stock effect dominating, as described by Mäler et al. (2003). In the other extreme we would have a watershed whose hydrological processes are not characterised by the phosphorus desorption processes (e.g., no hypoxia near sediment, low stocks of planktivorous fish), and/or whose turnover of water is relatively high (e.g. a river).
effects of this choice (0.1% in the Review) on the timing and intensity of optimal abatement efforts has been under extensive dispute (Stern et al., 2006; see Weitzman, 2007 for a critique). However, it is surprising that under certain conditions, this choice may even reverse the ordering of the amount of annual phosphorus losses associated with the private and social optimum.

There is also literature on whether farmers apply the market rate of discount in their decision making processes. For instance Schmitz (1995) and Weersink et al. (1999) analyze whether the farmers apply different discount rates for income granted by government policies and for sales revenues (with opposite results). Myyrä et al. (2007) postulate the land tenure insecurity to be reflected in higher discount rates and show that this leads to lowered investment levels on productivity, for instance on lowered phosphorus application rates.

The effect of the discount rate on optimal policies is substantial. The social rate of time preference is a subjective choice; but also the farmers' attitudes towards government payments (here: taxes on phosphorus and subsidies on VFS) are ambiguous. Altogether, these points suggest that farming activities should not be regulated 'automatically' at the intensive margin.

3 Empirical application

The four hectare sized parcels of the empirical application are all allocated on barley. The soil type is sandy clay (see Appendix 1 for a precise definition). Due to a linear damage function, the combination of parcel specific first-best optima will also be the optimum for the entire target area.

The parcels have two alternative, rectangular shapes: square and narrow. The ratio of field edges is 1:3 for the narrow parcel and 1:1 for the square one. The slope which is uniform along each parcel obtains a value of either 2% or 7%.

3.1 Functional forms

The explicit functions presented in the following are derived or calibrated for Finnish conditions. Sensitivity analysis will illustrate the generality of the empirical results. The functions and parameter values are presented collectively in Appendix 3.

Phosphorus response function
One of the few studies isolating and reporting a phosphorus response function is Myyrä et al. (2007) who base their specification on the long term fertilizer trials by Saarela et al. (1995):

\[
Y(s, x) = 3367(1 - 0.74e^{-0.37s}) + (21.7 - 0.414s)\sqrt{x} + (17.01 - 0.132s)\frac{s}{x} + 6.97
\]  

(13)

The first part in (13), \((3367(1 - 0.74e^{-0.37s}))\), is the Mitscherlich function which is increasing and concave in \(s\). The remaining terms capture the one period response on phosphorus fertilization. This effect is stronger for low STP soils.

**Phosphorus loss function**

We model the phosphorus loss \((L)\) as a sum of PP and DP losses \((L_{PP}\) and \(L_{DP}\)) where the former is weighed according to its bioavailability \((\zeta)\):

\[
L = \zeta L_{PP}(l, b, \gamma) + L_{DP}(s),
\]  

(14)

where \(l\) is the length of the lower field edge, \(b\) is the VFS width and \(\gamma\) is the slope of the parcel. Following Ekholm et al. (2005) we set \(\zeta = 0.16^4\).

Several studies have estimated the relationship between STP and DP loss. (see, e.g., Sharpley, 1995; Pote et al., 1996; Uusitalo and Jansson, 2002). Schroeder et al. (2004) provide a brief list of studies examining the relationship between easily soluble topsoil phosphorus, measured by STP, and the DP loss in runoff. We use the laboratory results of Uusitalo and Jansson (2002) to estimate DP concentration (mg l\(^{-1}\)) in runoff with the help of STP level: \(DP_{\text{concentration}} = 0.021s - 0.015\). We assume that the DP concentrations are identical in drainage flow and surface runoff. For the average annual runoff volume we use the value of 270 mm, adopted from Ekholm et al. (2005). Hence, the DP loss in kilograms becomes: \(L_{DP} = 0.0567s - 0.0405\).

The PP loss function in field surface captures the effect the slope has on erosion and the offsetting and filtering effects of a VFS. The function estimating the PP inflow entering a VFS is adopted from Uusitalo et al. (2007):

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\[ PP \text{ inflow} = (0.38\gamma + 0.12)(1 - A), \quad (15) \]

where \( \gamma \) is the slope of the parcel. PP inflow is given in kg ha\(^{-1}\) a\(^{-1}\); and \( A \) is the VFS acreage. This function captures the offsetting effect of the VFS. Increasing its width increases the acreage which decreases the second term in (15) and hence the inflow into the VFS. The inflow captured in (15) is exposed to filtering effect of the VFS. The VFS abatement as a percentage of the inflow is derived from simulated and actual data in Iho (2007). Multiplying the inflow (15) with a pass through ratio derived from the percentage abatement yields the function describing the PP loss from field surface as determined by the slope and the VFS width:

\[
L_{PP_{\text{surface}}} = (0.38\gamma + 0.12)(1 - A)\left[1 - \frac{25.57\ln(b + 1)}{100}\right] \quad (16)
\]

The equation in brackets in (16) depicts the ratio of inflow passing through the VFS. It is decreasing and convex in \( b \), that is, the VFS filtering is increasing and concave in VFS width.

Some of the PP loss takes place in the drainage flow not affected by the VFS. In Kniesel and Turtola (2000), the average amount of eroded material in the drainage was 567 kg ha\(^{-1}\), annually. Using a constant value for P concentration in soil (0.14%, Ekholm et al., 2005), the PP-loss in drainage from one hectare becomes: \( L_{PP_{\text{drain}}} = 0.79 \). This value is insensitive to field slope. Hence, the steeper the slope, the smaller the relative PP loss via the drainage.

Summing up the surface and drainage PP losses yields the PP loss from a hectare-sized parcel. Multiplying it with the bioavailability coefficient commensurates it with the DP loss.

Collecting the elements completes the phosphorus loss function:

\[
L = 0.16 \left[(0.38\gamma + 0.12)(1 - A)\left[1 - \frac{25.57\ln(b + 1)}{100}\right] + 0.79\right] + 0.0567s - 0.0405 \quad (17)
\]

We can verify that the partial derivatives of (17) have the desired properties.
**Damage function**

The environmental damage in our model is related to annual loss of bioavailable phosphorus. The damage function specification is adopted from Gren and Holmer (2003) who estimate the (constant) marginal damage from a kilogram of nitrogen load to be 6.6 € for the Baltic Sea countries. Using a Redfield ratio of 7.2, this transfers to a marginal damage of 47 € per phosphorus kilogram:

\[ D = 47L, \]

where the damage is in € kg\(^{-1}\).

**The vegetative filter strip cost function (f)**

The opportunity cost of land is the most important source of VFS costs. The rest comprises of management costs from mowing and baling the vegetative cover of the VFS. Palva and Peltonen (2006) estimate time and machinery requirements for two 15 meters wide VFSs.\(^6\) Based on their estimates, we assume the time requirement for a hectare of VFS to be 3.5 hours with the tractor plus an hour for preparation. For the hourly cost of machinery and labor we use the lowest market price for tractor work, including the driver. In Finland, this was 21 € in 2004 (Pentti and Laaksonen, 2005).\(^7\) We obtain:

\[ f = 94.5A, \]

where the cost is given in € ha\(^{-1}\).

**Transition function**

Equation 20 depicts the definition of phosphorus surplus:

\[ P_{\text{surplus}} = x - \delta(s)\eta Y(s,x), \]

where \(\delta(s)\) is the phosphorus concentration of the dry matter crop yield \(Y\) and parameter \(\eta\) obtains a value of 0.86. From Saarela et al. (1995) we can derive an estimate for \(\delta\).

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\(^5\) The variation between the periods is significant: in 1987- May 1991 the average PP-loss in drainage was 258 kg ha\(^{-1}\), and in July 1991-1993 1122 kg ha\(^{-1}\).

\(^6\) The costs per acreage would be very different for narrower strips. However, constructing VFS in the present scale is a fairly new phenomenon in agriculture. However, data on actual VFS management costs is poorly available.
\[ \delta = 3.49 + 0.216 \ln(s), \quad (21) \]

where the concentration is given in per mille. The relative phosphorus content is increasing and concave in STP. For instance with STP level 7 it is 3.91‰, and with STP of 12 it is 4.03‰. We modify the transition function of Ekholm et al. (2005) to capture the transition process of STP from period \( t \) to \( t + 1 \):

\[ s_{t+1} = 0.9816 s_t + (0.0032 + 0.00084 s_t) P_{\text{surplus}}, \quad (22) \]

The function by Ekholm et al. (2005) initially describes longer time periods (10-15 years). Due to the stationarity of our model we can use (22) as our transition function.

4 The results

4.1 The private and social optima

Table 2 presents side by side the privately and socially optimal steady state phosphorus fertilization and STP levels, VFS widths, and the associated steady state outcomes: the crop yield, the profits and the phosphorus loss from each parcel type.

Table 2. The private and social optima.

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<thead>
<tr>
<th></th>
<th>Private optimum</th>
<th>Social optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slope (%)</strong></td>
<td>1:1 1:3 1:1 1:3</td>
<td>1:1 1:3 1:1 1:3</td>
</tr>
<tr>
<td><strong>Shape</strong></td>
<td>2 7 2 7 2 7 2 7</td>
<td>2 7 2 7 2 7 2 7</td>
</tr>
<tr>
<td><strong>Fertilization (kg ha(^{-1}))</strong></td>
<td>27.1 27.1 27.1 27.1</td>
<td>26.3 26.3 26.3 26.3</td>
</tr>
<tr>
<td><strong>STP (mg l(^{-1}))</strong></td>
<td>9.0 9.0 9.0 9.0</td>
<td>8.0 8.0 8.0 8.0</td>
</tr>
<tr>
<td><strong>VFS (m)</strong></td>
<td>0 0 0 0 0 0 0.3 1.3</td>
<td>0 0 0.3 1.3</td>
</tr>
<tr>
<td><strong>Crop yield (kg ha(^{-1}))</strong></td>
<td>3419 3419 3419 3419</td>
<td>3386 3386 3374 3360</td>
</tr>
<tr>
<td><strong>One period profits (€ ha(^{-1}))</strong></td>
<td>334.4 334.4 334.4 334.4</td>
<td>328.1 328.1 326.7 324.9</td>
</tr>
<tr>
<td><strong>Phosphorus loss (kg ha(^{-1}))</strong></td>
<td>0.74 0.74 1.04 1.04</td>
<td>0.68 0.68 0.95 0.89</td>
</tr>
</tbody>
</table>

\(^7\) We assume that the lowest price offer is the best approximation for the true costs of VFS management. The average price was 25.2€.
The key results in table 2 are: 1) the VFSs are wider on steep and narrow parcels 2) the differences in optimal STP levels are insignificant between the parcels.

The privately optimal annual steady state phosphorus fertilization is 27.1 kg ha\(^{-1}\) for all parcels. This is associated with a steady state STP value of 9.0. The annual crop yield associated with these is about 3420 kg ha\(^{-1}\) and the annual profit equal to about 334 € ha\(^{-1}\). The phosphorus loss from the steeper parcel is about 41% higher then from the gentler parcel. Under the private optimum, the total phosphorus loss is 3.56 kg.

The socially optimal phosphorus fertilization varies between 26.3 kg ha\(^{-1}\) and 26.1 kg ha\(^{-1}\).\(^8\) The associated STP level is about 8.0 mg l\(^{-1}\) for each parcel. The crop yields are lower than at the private optimum, as well as the one period profits. For the gentler parcel, the socially optimal VFS width is zero. For the steeper parcel, there will be a VFS in the optimum; 0.3 meters and 1.3 meters wide for the square and the narrow parcel, respectively. The difference between the phosphorus losses associated with the private and the social optimum is the highest for the steeper parcels.\(^9\) There, the abatement (both absolute and relative) from the narrower parcel is higher than from the square parcel. The total abatement is 9.9 %.

The strip is wider for the narrow parcel because the acreage lost from production is smaller per unit of VFS width. Hence, the cost of a unit of VFS will be lower.\(^10\) The initial phosphorus loss in surface runoff depends only on the slope of the parcel, not on its shape; and the VFS abatement is increasing (though concave) in VFS width. Therefore, it is optimal to construct wider strips on narrower parcels.

The moderateness of the variation in optimal STP levels is notable. The maximum difference in the optimal levels for various parcel types is about 0.01 mg l\(^{-1}\) (0.1%). At the same time there are clear differences in optimal VFS widths. These differences are interrelated: the slight decreases in STP levels are associated with increasing VFS width. There is no physical link in our model between the DP losses stimulated by the STP levels, and the PP losses stimulated by erosion. Hence, this

\(^8\) The unit ha\(^{-1}\) should be interpreted as parcel\(^{-1}\) because the application varies within the hectar; the land allocated on VFS is not fertilized. The fertilization intensity per hectare under cultivation varies only little; the intensities are identical up to third integer.

\(^9\) For the rest of the paper, this difference in phosphorus losses associated with two steady states is called as abatement.

\(^10\) This is in accordance with the heuristic argument that the VFS should be wider at fields with longer upslope fields.
connection is due to substituting the PP loss abatement with DP abatement. This effect is minor because lowering STP makes the VFS less expensive by lowering the opportunity cost of land.

4.2 Sensitivity analysis

Our results move in expected directions as we vary the basic parameters. Higher output prices increase fertilization substantially. This increases the opportunity costs of land and reduces the VFSs in the social optimum. Input price variation, on the other hand, has milder effects, as it is not the sole source of costs. Increasing the marginal damage places more weight on phosphorus losses. That is, it decreases the socially optimal input use and increases the optimal VFS widths.

We analyze in more detail the sensitivity towards the salient elements of the model captured by the discount factor, the bioavailability of the PP loss and the parameters of the transition function. These are examined one at the time keeping other variables unchanged. We report either the social optimum or both optima, depending on the case. For clarity, we analyze only the narrow parcel with a 7% slope.
**Discount rate**

We allow the discount rate to obtain the values 1%, 3%, 5% (default value), 7% and 9%. The graphs in figure 1 depict the socially and privately optimal STP levels, as well as the associated abatement percentage. The privately (socially) optimal STP levels are denoted by STP p (STP s).

*Figure 1. Privately and socially optimal soil phosphorus under varying discount rates.*

![Discount rate graph](image)

<table>
<thead>
<tr>
<th>Discount rate</th>
<th>STP p</th>
<th>STP s</th>
<th>Abatement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>11.9</td>
<td>9.7</td>
<td>18.4</td>
</tr>
<tr>
<td>3%</td>
<td>10.2</td>
<td>8.8</td>
<td>15.7</td>
</tr>
<tr>
<td>5%</td>
<td>9</td>
<td>8.1</td>
<td>14.6</td>
</tr>
<tr>
<td>7%</td>
<td>8.3</td>
<td>7.5</td>
<td>14.2</td>
</tr>
<tr>
<td>9%</td>
<td>7.6</td>
<td>7</td>
<td>13.9</td>
</tr>
</tbody>
</table>

Figure 1 illustrates how the STP level is influenced by the discount rate. Changes in the discount rate change investments in alternative capital forms until the costs (benefits) of adaption are equal to marginal changes in the stationary rate of return to capital. In our case, higher discount rates lower the stationary STP levels and the associated phosphorus losses. Both absolute and relative drops in privately optimal STP are higher than those of the socially optimal STP. Also, the privately optimal STP levels associated with high discount rates are distinctly lower than the socially optimal STP levels associated with low discount rates.

As the discount rate increases, also the socially optimal VFS increases from 1.2 to 1.3 meters (not reported in figure 1). This has an opposite effect on abatement percentage as does the change in
STP levels. As the discount rate increases, the difference between the phosphorus losses associated with the private optimum and the social optimum decreases. On the other hand, a decrease in STP levels decreases opportunity costs which makes the VFSs less expensive.

**Bioavailability of PP**

The VFSs filter only PP loss from surface runoff. Therefore, the bioavailability of PP affects their optimal allocation. We allow the bioavailability parameter to obtain the values 0.06, 0.16 (the default value), 0.26, 0.36 and 0.46. Uusitalo et al (2003) have estimated the PP bioavailability under aerobic and anaerobic conditions. According to their results (for soils with STP ranges close to ours), 0.46 is a good upper bound for a PP bioavailability estimate.

*Figure 2. The effect of bioavailability of PP on socially optimal phosphorus policies.*

![Graph showing the effect of bioavailability of PP on socially optimal phosphorus policies.](image)

The optimal VFS widths and abatement percentages increase as the bioavailability coefficient increases. Also, the STP level remains practically unaltered (the increase of 0.3% not distinguishable in 2). The former is due to PP getting more harmful for water quality. The latter is due to the main determinants of the steady state STP level being unaltered: the input and output prices and the discount rate. The low interdependence of PP and DP loss control is logical. A marginal decrease in STP lowers the private profits making VFSs less expensive. But at the same
time, constructing VFSs gets less necessary as the total phosphorus loss is decreased due to lower STP level.

The policy implications are clear. If the receiving water body is sensitive towards PP losses, the erosion control measures should be used intensively. Conversely, if the PP losses do not affect the water quality, it is not worthwhile to allocate land away from production to mitigate erosion (more than is privately optimal).

Transition dynamics
The phosphorus carryover dynamics has substantial effects on the results. Were there no carryover, the optimization problem would be static; were the carryover perfect, we would need to use fertilizers only once and the stationary level would be zero (in this case there could be no crop uptake). We analyze the sensitivity of the results towards changes in carryover dynamics by varying the first parameter of (22)\(^1\). It determines the fraction of plant available soil phosphorus carried over to the next one. It obtains the values 0.9636, 0.9726, 0.9816 (default), 0.9906 and 0.9996. The value 0.9636, for instance, means that 3.64% of current period's plant available soil phosphorus reserves will be depleted between two periods, with a zero phosphorus balance.

\(^1\) We do not take stance towards how and why permanent changes in transition dynamics might occur.
Figure 3. Privately and socially optimal phosphorus use under varying carryover capacities.

Figure 3 depicts the values for the socially and privately optimal STP levels, and the annual fertilization associated with the privately optimal STP. As the carryover fraction of STP decreases, the stationary STP level decreases while the fertilization increases. The latter outweighs partly the effect of intensified phosphorus depletion. If the carryover fraction increases, both the farmers and the social planner optimally decrease fertilization levels and allow the STP level to increase.

These changes have only minor effects on abatement levels. The slight increase in the difference of the privately and socially optimal STP levels is outweighed by a slight increase in VFS widths. The abatement level varies between 14.6% and 15%. However, when moving from the lowest to the highest socially optimal STP level, the associated absolute phosphorus loss increases 12.4%, from 0.85 kg ha\(^{-1}\) to 0.95 kg ha\(^{-1}\). Hence, the higher the carry over capacity, the higher the STP levels and phosphorus losses under both private and social optimum.
4.3 Instrument design

In the following, we quantify some properties of propositions one and two. That is, we analyze how differences in discount rates affect optimal tax rates; and how spatial heterogeneity in carryover capacity affects the efficiency of taxes levied on alternative bases. We also provide some quantitative illustration on VSF subsidies.

Optimal VFS subsidies depend on the STP level. They are, however, independent of the tax base; both first-best taxes provide first-best STP levels. There are infinitely many subsidy functions that satisfy the first-best conditions at the margin. To guarantee uniqueness of VSF choice under subsidy, a subsidy function must be concave in VFS width. Because costs are linearly increasing in width, concavity assures that all units preceding the optimal VSF width provide the farmer with positive marginal net benefits. According to Lichtenberg (2002), a uniform instrument may achieve social optimum if the (heterogeneous) parcel type does not affect the environmental damage. Because the PP entering the strip (equation 15) depends on the slope, the first-best VFS subsidy must be differentiated.

The first-best VFS allocations and marginal subsidies are given in table tapiir15.

Table 3. First-best marginal VFS subsidies.

<table>
<thead>
<tr>
<th>Slope (%)</th>
<th>2</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape</td>
<td>1:1</td>
<td>1:3</td>
</tr>
<tr>
<td>Marginal subsidy (€ m⁻¹)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VFS width</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The marginal subsidy for the narrow parcels is lower than for the square parcel despite the wider VFS. The marginal costs of VFS abatement, however, are identical. For the narrow parcel a unit of width sets less land out of production. To equalize marginal abatement costs, the marginal subsidy for the narrow parcel must therefore be lower. The marginal subsidy is zero for parcels with no VFS.

12 Due to the stationarity of the analysis we do not assess the efficiency of various second-best instruments.
Alternative tax bases

We illustrate the effect of the tax base by allowing the carryover capacity to differ between the parcels. This difference is captured in the same parameter that we varied in the sensitivity analysis. For two of the parcels, the parameter obtains the value 0.9636 and for the other two the value 0.996. The social planner, however, assumes that all four parcels obtain the default parameter value 0.9816 and sets both taxes accordingly.\textsuperscript{13} Equation (12) defines the marginal tax levied on STP: \( \tau_s = D_s \approx 2.7s^4 \€ \) which is a constant tax rate. The tax levied on fertilizer use is given by (10):

\[
\tau_s = \beta \Gamma_s \frac{D_s}{(1 - \beta \Gamma_s)}.
\]

Table 4 presents the privately and socially optimal STP levels for the two parcel types. Also, it presents the STP values as farmers' optimal responses when facing either of the taxes; and the associated phosphorus losses.

<table>
<thead>
<tr>
<th>Table 4. Alternative tax bases and heterogeneity in carryover capacity.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition parameter</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>0.9636</td>
</tr>
<tr>
<td>0.9996</td>
</tr>
<tr>
<td>Phosphorus loss (kg)</td>
</tr>
</tbody>
</table>

Table (4) shows that the first-best optimum is attainable even when the social planner has incorrect information on transition dynamics - if the tax is levied on STP. If the tax is levied on fertilization, the first-best optimum is not achieved. The low (high) carryover capacity parcels are facing too high (low) a tax and the associated STP levels are below (above) the social optimum. Also, the phosphorus loss from the area is above the social optimum.\textsuperscript{14}

Difference in discount rates

Proposition two stated that a difference in the discount rates applied by the farmer and the social planner will alter the optimal stationary marginal tax. We analyze how increasing this difference will affect tax levied on STP. We conduct the analysis for a parcel with socially optimal VFS width equal to zero.

\textsuperscript{13} For clarity, we set the bioavailability coefficient to 0.06 so that the social optimum has no VFSs on any parcel.
If the social planner applies a discount rate of 5% (discount factor: 0.952), the marginal tax is given by:

\[
\tau_s = \frac{0.952}{\beta_p} D_s - \frac{\pi_s \Gamma_s (\beta_p - 0.952) + \pi_s \Gamma_s (0.952 - \beta_p)}{\beta_p \Gamma_s}
\]  

We allow the privately applied discount rate to increase to 9% while holding the socially applied discount rate at 5%. The optimal tax rate is depicted in figure 4. The vertical axis denotes the optimal steady state tax on STP and the horizontal axis the discount rate of the farmer.

Figure 4. Tax on STP when discount rates applied by the farmer and the social planner differ.

The dotted line depicts the discount rate for which the marginal tax rate is zero. The interception of the graph and the vertical axis denotes the tax rate when both discount rates are equal to 5%, i.e. 2.7 €. For farmer’s discount rates higher than 5%, \( \tau_s < 2.7 \) €. If the discount rate is 7.6, the marginal tax

\[\text{Even the private revenues are slightly higher under the tax on STP.}\]
is zero. Here, the socially and privately optimal choices of long term phosphorus fertilization coincide. The farmer does not acknowledge the environmental damage, but higher impatience makes the optimal STP levels identical. For discount rates above 7.6, the farmer must be subsidized to increase the fertilizer use to the socially optimal level.\textsuperscript{15}

5 Discussion

We analyzed spatially optimal steady state phosphorus policies in cereal production. We solved the socially and privately optimal steady state choices of phosphorus fertilization and vegetative filter strip (VFS) construction using dynamic programming. We also examined first-best instruments to sustain the social optimum. The arguments of our phosphorus response function were phosphorus fertilization and the plant available soil phosphorus, approximated by STP. We modelled phosphorus loss as a sum of dissolved, readily bioavailable phosphorus (DP) and particulate phosphorus (PP). The former was determined by the STP level and the latter by erosion. The susceptibility for erosion was sensible towards the slope of a parcel.

In the empirical application we found the socially optimal steady state phosphorus fertilization levels to be about 0.8 kg ha\textsuperscript{-1} lower than the privately optimal ones. The stationary privately (socially) optimal STP levels were 9.0 (8.0). There was minor variation in the optimal STP levels between the parcels, depending on the optimal VFS width. It was also optimal to construct VFSs on steeper (7\%) parcels but not on the gentler ones (2\%). The optimal VFS was wider on the narrow parcel than on the square formed. The phosphorus loss associated with the social optimum was about 10\% lower than the one associated with the private optimum.

Our first-best tax-subsidy schemes included a Pigouvian tax based directly on the STP level. The analysis revealed novel features, for instance, using the level of soil phosphorus as a tax base was analytically shown to require less information from the social planner than if the tax would be based on the annual fertilization. Also the difference in the discount rates applied by the farmer and the social planner had interesting effects on first-best taxes. If, for instance, the discount rate applied by the farmer (social planner) would be 7.3\% (5.3\%), the optimal tax would be zero. The analysis on optimal taxation was summarized in propositions one and two.

\textsuperscript{15}This rather peculiar situation might be realistic, for instance, in the case of tenant farmer with a contract for a fixed time period. However, this case fits poorly our infinite time horizon framework. For a study on the effects of tenant uncertainty on phosphorus use, see Myyrä et al. (2007).
Our model was deterministic and stylized. However, the results on optimal taxation generalize to practical policy recommendations. It might be beneficial to condition the agri-environmental phosphorus policies targeted on cereal production directly on STP. In many countries, regular soil tests are already conducted. In Finland, for instance, the ongoing agri-environmental program conditions the allowable fertilization ranges on the measured STP levels. But beyond controlling nitrogen applications, what is the need for controlling fertilization if the environmental damage (and most of the crop response) is due to accumulated soil phosphorus? Furthermore, it is sometimes argued that certain field parcels need higher phosphorus fertilization in the beginning of the season, and that the extra fertilization is uptaken by higher crop yield. This suggests that the transition dynamics varies between (and even within) the parcels. Allowing the farmers to choose the phosphorus fertilization levels and controlling only the realized STP levels might be beneficial for both the environment and for the farmers.

Also, the analysis on discount factors generalizes to practical agri-environmental policy. Public opinion - and the authorities - tends to require specific abatement levels from each industrial sector. Eventually, this translates into abatement requirements concerning individual parcels. Our analysis shows, however, that the phosphorus loss levels at the intensive margin might be even too low from the viewpoint of social welfare. It might be that the intensive margin input use is too low, and that the reductions would be efficiently posed on the extensive margin. That is, it might be beneficial for the environment and for the economy to allow for increasing fertilization intensities and to restrict the total area allocated on agriculture. However, the analysis on the extensive margin is beyond the scope of this study.

An obvious extension to this study would be the path analysis. This would allow for comparing the efficiency properties of various second-best instruments. In particular the optimal design of VFS subsidies would be interesting. Erosion is strongly dependent on observable parcel characteristics, slope in particular. How much would we gain by conditioning the subsidies on these characteristics and not only on VFS width?

It would also be interesting to include stochasticity of prices and/or natural conditions to see whether these would alter, say, the relative shares of PP and DP abatement. Their influence on the optimal accumulation of soil phosphorus might also be worth examining.
Acknowledgements

I am indebted to Markku Ollikainen and Marita Laukkanen for their valuable comments. I have benefited from the comments of the participants at the EAAE Congress in Copenhagen, at the EAERE Conference in Thessaloniki and at the Environmental Economics Colloquium in Helsinki University. The article is a part of the project Non-Point Source Pollution Economics (NOPEC) funded by Academy of Finland (no. 115364). During the writing process, I have also enjoyed financial support from the Finnish Cultural Foundation, and the Kyösti Haataja Foundation.
References


Appendices

Appendix 1. The soil type of the target area.

This soil type is a typical Finnish clay soil. It consists of the following textures:

<table>
<thead>
<tr>
<th>Particle size fraction</th>
<th>Percentage value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay (&lt;0.002mm)</td>
<td>30-60</td>
</tr>
<tr>
<td>Silt (0.002-0.02mm)</td>
<td>&lt;20</td>
</tr>
<tr>
<td>Fine sand (0.02-0.2mm)</td>
<td>20-70</td>
</tr>
</tbody>
</table>

Appendix 2. The costs of farming.

Following Myyrä et al. (2003), we calculate the prices of fertilizers from the prices of fertilizer mixes (www.tigoteam.com, cited 23.1.2008). Depending on the mix, applying a fixed amount of nitrogen per hectare requires different total amounts of fertilization. Hence, for fixed nitrogen intensity, each mix is associated with a different amount of phosphorus. We regress this amount of phosphorus against the cost of fertilization per hectare. The obtained slope denotes the computational price of P - which is actually independent of the amount of N applied. The constant term in regression denotes the other fertilization costs, which contain here also the costs of nitrogen and potassium. This term is determined by the amount of nitrogen application, assumed here to be 90 kg/ha. Calculated this way, the price of P is 1.68 € kg\(^{-1}\) and the (other) fertilization costs are 104 € ha\(^{-1}\). In addition to fertilization costs, the following fixed and variable costs were included in \(C\), adopted from Lankoski et al. (2006).

<table>
<thead>
<tr>
<th>Cost category</th>
<th>€ ha(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs</td>
<td>138.9</td>
</tr>
<tr>
<td>Variable costs:</td>
<td></td>
</tr>
<tr>
<td>Tractors</td>
<td>19.5</td>
</tr>
<tr>
<td>Machinery</td>
<td>22.4</td>
</tr>
<tr>
<td>Labour</td>
<td>50.8</td>
</tr>
<tr>
<td>Seeds</td>
<td>40</td>
</tr>
<tr>
<td>Fertilization (not P)</td>
<td>104</td>
</tr>
<tr>
<td>Total costs</td>
<td>375.6</td>
</tr>
</tbody>
</table>
Appendix 3. The empirical functions and parameter values.

<table>
<thead>
<tr>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P response (kg ha(^{-1}))</strong></td>
</tr>
<tr>
<td><strong>Phosphorus loss (kg ha(^{-1}))</strong>;</td>
</tr>
<tr>
<td>• PP, surface (kg ha(^{-1}))</td>
</tr>
<tr>
<td>• PP, drainage (kg ha(^{-1}))</td>
</tr>
<tr>
<td>• DRP loss (kg ha(^{-1}))</td>
</tr>
<tr>
<td><strong>Transition function</strong>;</td>
</tr>
<tr>
<td>• P surplus (kg)</td>
</tr>
<tr>
<td>• P concentration (‰)</td>
</tr>
<tr>
<td>Damage (€ kg(^{-1}))</td>
</tr>
<tr>
<td>VFS costs (€ ha(^{-1}))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>price, barley (€ kg(^{-1}))</strong>(^{16})</td>
</tr>
<tr>
<td><strong>price, phosphorus (€ kg(^{-1}))</strong></td>
</tr>
<tr>
<td><strong>constant costs (€ ha(^{-1}))</strong></td>
</tr>
<tr>
<td><strong>discount factor</strong></td>
</tr>
<tr>
<td><strong>dry matter ratio</strong></td>
</tr>
<tr>
<td><strong>bioavailability coefficient</strong></td>
</tr>
</tbody>
</table>

\(^{16}\) We use an average price of barley for fodder (0.187€ kg\(^{-1}\)) and for malt (0.255€ kg\(^{-1}\)).
Discussion Papers:

No.


